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AN ANALYTICAL STUDY OF SOME PROBLEMS IN PARTIAL DIFFERENTIAL EQUATIONS WITH APPLICATIONS TO FLUID DYNAMICS AND WAVE PROPAGATION

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SUMMARY OF RESEARCH

Much of the research effort during the grant period was devoted to investigations [1;6-9] of periodic waves, and stability in fluid dynamics. Progress was also made [2-5] on marching schemes in acoustic wave propagation. (Numbers in brackets refer to the publications listed after this summary.)

In the area of fluid dynamics, problems relating to spiral flows of a viscous incompressible fluid between concentric cylinders in the narrow gap limit were investigated (with D. Sather). When the motion of the fluid is driven by rotating and sliding of the cylinders, the problem is that of rotating, plane, Couette flow (RPCF); when an axial pressure gradient is added, rotating, plane, Couette-Poiseuille flow (RPCPF) ensues. Various such problems for viscous, incompressible flow lead to abstract equations of the form

$$(*) \qquad \frac{du}{dt} = Lu + N(u), \qquad u \in B,$$

in a Banach space B. Here L is a linear operator depending on Reynolds number, λ, and a structural parameter γ. N is a nonlinear operator. For the problem of RPCF, we determine in [1;6;7] for each fixed value of γ near $\gamma = 0$ a unique branch of periodic orbits of $(*)$ that bifurcates supercritically from the steady primary flow at a critical value $\lambda_c(\gamma)$ (in this problem γ measures the difference between the spiral directions of the primary and secondary flows). These solutions are seen to represent periodic waves. It is frequently the case in bifurcation problems that one makes assumptions (e.g.,on the signs of certain coefficients involving nonlinear functionals) necessary for the stability and supercritical behavior of bifurcating states; here we are actually able to verify such a condition by evaluating certain cubic integrals involving the nonlinear operator. These results for RPCF suggest the existence of a continuum of such periodic orbits depending continuously on γ. In [9] this continuous dependence is proved. The continuity as $\gamma \to 0$ is not routine and involves a complicated analysis of a singular Hopf bifurcation problem. The existence of such a continuum of states, each of which is already fairly complicated, offers a possible explanation of turbulence, since imperfections in physical apparatus may lead to variations in γ over time.

The techniques of [6;7;9] for RPCF are extended to related problems, partly in [6] and especially in [8], where the RPCPF problem is considered. Then L in $(*)$ has a more complicated structure and depends on three parameters measuring the various Couette and Poiseuille forces. When the Poiseuille forces are dominated by the Couette forces, the existence of stable periodic waves was proved in [8]. (A preliminary study of the problem of Langmuir circulations in upper ocean mixing indicates a strong relation to models considered in [1;6-9]; this connection is being investigated in current research.)

In the area of acoustic wave propagation, schemes were developed and tested (with D. F. St.Mary and G.-Q. Li) [2;3;4;5] for propagating forward in range the solutions of elliptic and parabolic equations associated with ocean acoustics. Although initial-value problems for elliptic equations are known to be ill-posed, it was demonstrated in [2;3;5] that a numerical scheme based on such an elliptic problem (for a far-field Helmholtz equation) can be stable if appropriate restrictions are observed on the stepsizes. These restrictions are derived in [5] by a careful matrix stability analysis. Calculations in [5] on a model problem show the ability of the elliptic marching method to account for backscatter if the starting data are sufficiently well known. In [4] two approaches are presented for Crank-Nicolson schemes for a parabolic system obtained from the elastic wave equations in the presence of a liquid/solid interface. The elliptic marching technique developed in [2-5] also appears to succeed for the elastic problem.

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PUBLICATIONS DURING THE GRANT PERIOD

[1] Waves in rotating plane Couette flow. To appear in *Geometry, Topology and Number theory,* G.M. Rassias, Ed., World Scientific, Singapore, 1990 (with D. Sather).

[2] Computational ocean acoustics, Proceedings of the second Edward Bouchet International Conference on Physics and Technology, L. E. Johnson & J. A. Johnson (eds.), U. of Ghana, Legon, 98-108, 1990. (with G.-Q. Li and D. F. St. Mary)

[3] Marching techniques based on elliptic wave equations, Proceedings, IMACS '91 13th World Congress on Computational and Applied Mathematics, R. Vichnevetsky and J. J. H. Miller (eds.), Trinity College, Dublin, **2**, 537-538, 1991. (with D. F. St. Mary)

[4] Numerical solutions of the elastic wave equation, Proceedings, IMACS '91 13th World Congress on Computational and Applied Mathematics, R. Vichnevetsky and J. J. H. Miller (eds.), Trinity College, Dublin, **2**, 543-544, 1991. (with G.-Q. Li and D. F. St. Mary)

[5] Stable marching schemes based on elliptic models of wave propagation. Submitted to J. Acoust. Soc. Amer., 1991 (with D.F. St. Mary).

[6] Time-periodic states in problems containing a structure parameter. To appear in Proceedings of the First European Conference on Elliptic and Parabolic Problems, Pont-à-Mousson, June, 1991. (with D. Sather)

[7] Periodic waves in rotating plane Couette flow. To appear in Z. Angew. Math. Phys.,1992 (with D. Sather).

[8] Symmetry in rotating plane Couette-Poiseuille flow. Submitted to Amer. Math. Soc., Lects. in Appl. Math., 1992 (with D. Sather).

[9] Continua of periodic waves in rotating plane Couette flow. Submitted to J. Differential Equations, 1992 (with D. Sather).